Statistics of multispacecraft observations of chorus dispersion and source location

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[1] We report emission characteristics of 52 chorus events on 23 August 2003 and 10 events on three other days, modeled with a ray tracing technique. Chorus waves have a characteristic frequency/time variation that is a combination of frequency separation by propagation dispersion and a time-dependent source frequency emission drift. A cross-correlation technique comparing data from multiple Cluster spacecraft quantifies the frequency variation owing to propagation dispersion. The comparison of the data cross correlations with the simulated cross correlations allows the identification of a correlation region which has at least one common point with the chorus source region. Any remaining frequency/time variation in the single-spacecraft spectrograms not accounted for by the cross correlations is then used to determine the time-dependent source frequency emission drift. The final modeled correlation region and source frequency emission drift for each chorus event is consistent with both the cross-correlation and single-spacecraft data. The modeled correlation regions are located near the magnetic equator and are, in general, more extended parallel to the Earth’s magnetic field than perpendicular to it. It is found that waves with frequencies above and below 1/2 the equatorial electron cyclotron frequency on the magnetic field line of the spacecraft (lower and upper band, respectively) are emitted in a broad spectrum of wave normal angles. There is also some preference for lower band waves observed at the spacecraft to have been emitted near the Gendrin angle and at earthward-pointing wave normal angles of between −20° and −30°. The latter result is close to the range of wave normal angles shown recently to be connected with chorus that propagates into the plasmasphere and evolves into the incoherent plasmaspheric hiss spectrum, known to be connected to pitch angle scattering and loss of electrons in the electron slot region. Finally, the time-dependent source frequency emission drift for these events ranges from 1 to 20 kHz/s. For most events these rates account for at least 2/3 of the chorus frequency/time variation with the rest being due to propagation dispersion.


1. Introduction

[2] Chorus are intense electromagnetic wave emissions that propagate in the whistler mode and are observed in the magnetospheres of Earth, Jupiter [Gurnett et al., 1981; Inan et al., 1983], and Saturn [Reinleitner et al., 1984]. Electric field wave amplitudes at the Earth typically exceed ~0.5 mV/m [Meredith et al., 2001] and magnetic wave amplitudes are on the order of ~100 pT [Horne et al., 2005; Burris and Helliwell, 1975]. These large amplitudes are thought to be generated from a nonlinear process involving whistler mode waves interacting with 10–100 keV electrons via a cyclotron resonance mechanism [Kennel and Petscheck, 1966; Kennel, 1966; Kennel and Thorne, 1967; Trakhtengerts, 1995; Nunn et al., 1997; Katoh and Omura, 2007].

[3] Recent research suggests that chorus waves play an important role in the electron dynamics not only of the Earth’s magnetosphere, but also the plasmasphere. Summers et al. [1998], Meredith et al. [2003], and Horne et al. [2005] have shown that electron acceleration by chorus waves can increase the energetic electron flux in the radiation belts during the storm recovery phase by up to 3 orders of magnitude over prestorm values. These electrons are energetic enough to damage sensitive satellite components. In addition, recent ray tracing studies by Chum and Santolík [2005] and Bortnik et al. [2008] have shown that chorus waves emitted with a particular range of wave normal
angles are able to propagate into the plasmasphere where they evolve into a component of the observed equatorial whistler mode hiss spectrum. This hiss was shown by Thorne et al. [1973] and Lyons and Thorne [1973] to be an efficient pitch angle scatterer of electrons, leading to electron loss and the formation of the electron slot region.

Chorus waves are often seen as a succession of tones rising or falling in frequency with time on the order of a few tenths of a second. They are generally separated into a lower and upper band of emission delineated by a band of no emission at 1/2 of the equatorial value of the cyclotron frequency on the magnetic field line of the spacecraft observing the emission ($f_{ceq}$). In general the lower band exists in the frequency range of $f_{LHR} < f < f_{ceq}/2$, where $f_{LHR}$ is the lower hybrid frequency, and peaks in power at $ff_{ceq}/2 = 0.34$, whereas the upper band exists in the frequency range of $f_{ceq}/2 < f < f_{ceq}$ and peaks in power at $ff_{ceq}/2 = 0.53$ [Burits and Helliwell, 1976]. Wave normal angles of unducted chorus detected within a few degrees of the magnetic equator are usually confined to within 20° of the magnetic field for the lower band [Burton and Holzer, 1974; Goldstein and Tsurutani, 1984; Santolik et al., 2003] and can extend up to the resonance cone angle for the upper band [Hayakawa et al., 1984]. It has also been suggested that many lower band rays propagate at the Gendrin angle, defined as the nonzero wave normal angle that gives a group velocity along the Earth’s magnetic field [Goldstein and Tsurutani, 1984; Lauben et al., 2002]. Outside of a few degrees of the magnetic equator gradients in the density and magnetic field cause the average wave normal angle of both bands to increase from their equatorial values which indicates unducted propagation [Thorne and Kennel, 1967; Horne and Thorne, 2003; Santolik et al., 2004].

Recently, a few attempts have been made to explain the observation by Gurnett et al. [2001] that the frequency range and characteristic frequency/time spectral slope of a single chorus element can differ on spatially separated spacecraft. Inan et al. [2004] and Platino et al. [2006] have proposed that the different frequencies can be explained by a Doppler shift caused by a radiating source that is rapidly moving along the Earth’s magnetic field. The amount of Doppler shift depends on both the source velocity and the angular separation of the source to each spacecraft. These spectral characteristics have also been shown by Chum et al. [2007] to be consistent with a nonmoving source that emits radiation in a narrow beam of wave normal angles that rotates in time and sweeps in frequency.

2. Data and Cross-Correlation Technique

Chorus source regions have been modeled for 52 events on 23 August 2003, 5 events on 17 November 2003, 4 events on 27 November 2000, and a single event on 6 December 2003. Figure 1 shows successive chorus emissions (electric field wave power) in the lower band observed on the Wide Band Data instrument on each of the four Cluster spacecraft. Many of the same chorus events can be seen on all four spacecraft, including the event indicated by the arrow. All of the chorus activity simulated on this day exists between 2242 to 2302 UT. In this time the spacecraft
are at ~1300 MLT and travel northward toward the equator, from a magnetic latitude of ~20° to ~10° and earthward from an L-shell of 5.6 to 4.9. The separations between spacecraft (sc) pairs parallel to the Earth’s magnetic field are ~50 km for sc2–sc3; ~700 km for sc1–sc2, sc1–sc3, sc2–sc4, and sc3–sc4; and ~1400 km for sc1–sc4. The perpendicular separations are all less than 250 km. These separations are large enough to make a difference in ray propagation time between spacecraft and small enough for all four spacecraft to observe the same chorus events. The spacecraft relative separations and positions in the magnetosphere are comparable for the other 3 days. The results presented for these days are only for comparison with the 23 August 2003 events.

A single chorus element is often observed to have a different frequency/time slope on spatially separated spacecraft. This slope is due to both frequency dispersion by ray propagation and the increase or decrease in the frequency emitted by the source with time, termed the source frequency emission drift. While the source frequency emission drift is initially an unknown quantity, the amount of propagation dispersion can be determined by computing a cross correlation for each spacecraft pair. Figure 2 shows an example of a chorus emission seen on two Cluster spacecraft and the corresponding cross correlation which quantifies the time difference of arrival of wave power in each frequency channel from one spacecraft to the other. For each cross-correlation pair a region in space can exist where the simulated region for each spacecraft to be consistent with the correlation region. [9] The Doppler shift due to spacecraft motion has been estimated to be less than 30 Hz for the events discussed here and is negligible. We have chosen not to include either the moving source Doppler shift effects reported by Inan et al. [2004] and Platino et al. [2006] or the rotating source effects described by Chum et al. [2007].

3. Ray Tracing Method

All of the simulations used in this research utilize a ray tracing program described by Santolík et al. [2006] and developed from older programs written by Cerisier [1970] and Cairo and Lefeuvre [1986]. This ray tracing program uses a modified version of the ray tracing equations [Stix, 1992, p. 84] called Haselgrove’s equations [Haselgrove, 1955] which are better suited for a wide range of rays, including those with wave normal angles near the resonance cone. Rays are traced under the approximation of geometric optics, also known as the Wentzel-Kramers-Brillouin (WKB) approximation, which is valid when the scale over which the index of refraction changes significantly in the magnetosphere is larger than the wavelength of the wave. In the procedure of Santolík et al. [2006] the ray tracing equations are numerically integrated using a fourth-order Runge-Kutta method with one midpoint and adaptive integration step and the position and wave vector of the ray and are determined as a function of time. Rays are terminated if they change wave mode or cease to satisfy the WKB approximation. No attempt was made to simulate wave growth or damping.

Figure 2. (a and b) Chorus elements for a sample event as seen on two Cluster spacecraft. (c) Cross correlation between them. The common minimum and maximum frequencies \( f_i \) and \( f_h \), used as the frequency limits of the cross correlation, are shown by the dotted horizontal lines and correspond to a time difference \( \Delta t = t_h - t_i \) that is different on each spacecraft. The lines of minimum and maximum slope shown in each plot are the 1σ variations of the best fit line.

by artificially delaying the simulated emission of the higher frequencies relative to the lower frequencies. This represents a source frequency emission drift. As the delay is increased or decreased, the region in space where the simulated single-spacecraft data is consistent with the observed data changes. A proper source frequency emission drift is identified as one that allows the simulated region for each spacecraft to be consistent with the correlation region.
diffuse equilibrium model with the addition of a plasma-pause [Carpenter and Anderson, 1992]. The density is calibrated at the satellite location (marked by a cross) by plasma parameters from the WHISPER instrument onboard the Cluster spacecraft [Décraéau et al., 1997]. This model ignores ionization ducts and longitudinal density variations. The magnetic field is given by a simple dipole model scalable to match observed magnetic field values. For the L-values in question (L \sim 4–6) the Earth’s field is close to dipolar on the dawnside except perhaps in times of extreme magnetic activity. The models on the other 3 days are similar with the exception of the location of the plasma-pause on 27 November 2000 which is pushed earthward to an estimated value of L \sim 2.7 R_E owing to the increased amount of geomagnetic activity. The locations of the modeled correlation regions are not overly sensitive to the exact values of the above density and magnetic field models.

[12] The rays in this study are simulated only in the meridional plane (\phi = 0^\circ). A study by Burton and Holzer [1974] showed that rays are emitted into three-dimensional cones with the axis centered on the meridional plane. In the meridional plane all rays in the simulations are emitted at the location of a single spacecraft northward toward the equatorial chorus source region, that is, in the opposite direction from the actual propagation which is from source to spacecraft. Poynting flux measurements from the Cluster STAFF instrument have verified that the source regions are equatorward of the spacecraft on each day. The rays are emitted in wave normal increments of \Delta \theta_b = 0.05^\circ within a cone with a half angle from the magnetic field line that extends to a maximum angle of 5^\circ less than the resonance cone angle. Rays within 5^\circ of the resonance cone can be extremely sensitive to small changes in initial conditions and are not simulated. This reverse ray tracing technique is possible because the ray tracing equations are reversible in time, and it has the advantage over forward ray tracing of requiring no presupposition of the source region.

[13] In order to perform comparisons of ray values at different locations and for different frequencies all of the simulated rays were extrapolated to a discrete grid applied to the simulated magnetosphere. The grid resolution for each simulated frequency is \sim 50 km, which was chosen to optimize the simulation resolution as a function of computing time.

[14] For the chorus element shown in Figure 2 spacecraft 1 sees a larger maximum frequency than spacecraft 4, likely due to ray accessibility. The minimum and maximum frequencies are determined visually for each spacecraft and are simulated by emitting them in the previously described cone of angles. A difference in time of propagation between the minimum and maximum frequency on each spacecraft (\Delta t_{sim}) then exists at every location in the magnetosphere through which both rays propagate. This simulated time difference can be plotted as a function of frequency to find the frequency/time slope, which can then be compared to the single-spacecraft frequency/time slopes from actual data.

[15] To simulate the data cross correlation between two spacecraft (Figure 2c) the time difference of propagation between the common high frequency (f_H) on the two spacecraft is simulated, resulting in a propagation delay for the high frequency (d_{H-sim} = t_{H-sim} - t_{H-sim}) at every location in the magnetosphere through which the rays from the two spacecraft both propagate. This is repeated for the lower of the two common frequencies (f_L) which gives d_{L-sim} = t_{L-sim} - t_{L-sim}. The simulated time difference between the two delays can be plotted as a function of frequency to find the cross-correlation slope (\Delta d_{sim} = d_{H-sim} - d_{L-sim}). This can then be compared with the data cross correlation best fit slope from actual data.

[16] The modeled region for each individual data cross-correlation plot is identified as the area where the simulated cross correlation matches the data cross correlation within a 1 \sigma variation of the best fit slope. The spatial region of common overlap of all the individual modeled cross-correlation regions is identified as the correlation region. Since these cross-correlation time delays represent relative differences in ray propagation from one spacecraft to another they are independent of any source frequency emission drift.

[17] Now that the slope and dispersion are known and the correlation region has been found, the source frequency emission drift can be identified. For each single-spacecraft simulation an arbitrary source frequency emission drift is introduced by delaying the emission of the high frequency f_H relative to the low frequency f_L. The range of possible source frequency emission drifts is identified as those that cause the single-spacecraft simulations to overlap with the correlation region. The final modeled region matches both the cross-correlation data and the single-spacecraft data. For further detail on the method, see Breneman et al. [2007].

[18] Test simulations were conducted for a few test events using reduced bandwidths by varying the minimum and maximum frequency values simulated for each spacecraft. The correlation regions and source frequency emission drifts identified were found to be consistent to those determined by simulating the largest possible visually determined bandwidth. Using as large a bandwidth as possible, however, increases the number of frequency channels used in determining the best fit line for each chorus element, thereby reducing the uncertainty of the line.
fit and allowing a more precise determination of the correlation region and the source frequency emission drift.

4. Correlation Region Results

[19] The modeled correlation regions for 52 events on 23 August 2003 and 10 events on the other 3 days have been identified with the reverse ray tracing technique. (A video (Animation 1) of the correlation regions for all of the 23 August events is available.) Figure 4a shows an example correlation region modeled for a 23 August 2003 event. The spacecraft are identified by the cross and are on a trajectory indicated by the arrow. This is a fairly typical correlation region with respect to its distance from the spacecraft and topology. Correlation regions of this shape, more extended along the magnetic field line than across it, are often seen. Indeed, this is the most commonly determined shape. This is likely due to the ease of the rays propagating along field lines relative to crossing them, but the obtained shapes of the correlation regions do not necessarily reflect the true source topology. This is because the true source need only have a single point in common with the identified correlation region. Figure 4b shows histograms of the distances from spacecraft 1 to all 52 modeled correlation regions. Each bin on the histograms represents a span of 0.2 \( R_E \). These histograms indicate that chorus emissions detected on all four Cluster spacecraft are observed at an average distance of just under \( 1 R_E \) from the source. Corus emissions at this distance have a large enough dispersion that their cross correlations provide a useful identification of the correlation region. Figure 4c shows that the majority of modeled regions are located within 1 \( R_E \) of the magnetic equator which, at the source altitude of \( \approx 4.8 R_E \) corresponds to a maximum latitude of \( \approx 12^\circ \). The results for the other 3 days are similar.

5. Source Emission Characteristics

5.1. Wave Normal and Group Velocity Distributions

[20] Since the wave properties of lower and upper band rays have been observed in past studies to be somewhat different, the histograms for these two cases are presented separately. Figure 5 shows the cumulative histograms of each correlation region on 23 August 2003 for simulated wave normal and group velocity angle, separated by upper and lower band as determined from the estimated equatorial value of the cyclotron frequency on the magnetic field line of the spacecraft (\( f_{ceq} \)). Out of the entire set of rays simulated that includes all possible wave normal angles (except those within 5\(^\circ\) of the resonance cone) Figure 5 shows only the subset of discrete rays that satisfies the timing criteria described in section 3 at each grid point within the correlation region.

[21] Each bin represents a 5\(^\circ\) span in angle measured relative to the magnetic field. Negative values represent wave normal angles pointing earthward of the Earth’s magnetic field while positive values represent wave normal angles pointing antiearthward. Both upper and lower band histograms show a wide distribution of wave normal angles in the correlation regions. The lower band distribution shows peaks at wave normal angles of \( \approx 30^\circ \) and \( \approx 20^\circ \) to \( \approx 30^\circ \) while the upper band histogram shows only a single peak at \( \approx 30^\circ \) to \( 40^\circ \). The peak at \( \approx 50^\circ \) in the lower band represents rays that originate from the most commonly determined correlation region shape, an example of which is shown in Figure 4. The raypath of a 50\(^\circ\) ray for most of the simulated lower band frequencies is able to propagate along the entire length of one of these correlation regions. This can result in a single-source wave normal histogram that is sharply peaked at \( \approx 50^\circ \). Since each correlation region contributes equally to the overall wave normal histogram and there are many correlation regions of this shape the peak at \( \approx 50^\circ \) is quite pronounced.

[22] Group velocity angles for both bands point earthward and are largely confined to within a few degrees of the magnetic field, particularly for lower band waves. This suggests that many of the lower band waves at larger wave normal angles must be propagating near the Gendrin angle in order for their group velocity to be nearly aligned with the magnetic field. The results for the other 3 days are similar.

5.2. Source Frequency Emission Drift

[23] The modeled regions also yield information on the source frequency emission drift. Figure 6 shows example
spectrograms of two types of chorus elements: a constant slope chorus element (Figure 6a) and a variable slope chorus element (Figure 6b). Each is shown with the best fit line. The reduced chi-squared value is \( \chi^2_R = 1.3 \) for the constant slope chorus element on the left, which indicates a good line fit that accurately represents the data. The chi-squared value is \( \chi^2_R = 38 \) for the variable slope chorus element (concave downward) on the right, which indicates a bad line fit. This demonstrates that chorus elements of the first type can be fit to a line with a reasonable reduced chi-squared value while chorus elements of the second type cannot. Twenty-nine of the 52 events on 23 August 2003 were of the first type; a constant slope with a chi-squared value which indicates a good fit. The range of source frequency emission drifts was identified for each of these events. The remaining 23 events were of the second type and no attempt was made to identify the range of source frequency emission drifts.

Figure 6. (a) Chorus emission with a frequency that linearly increases with time with a reduced chi-squared value of \( \chi^2_R = 1.3 \) which indicates a good fit to the data. (b) Chorus emission with a frequency that does not linearly increase with time with a reduced chi-squared value of \( \chi^2_R = 38 \) which indicates that the line fit does not accurately represent the data.

6. Discussion

6.1. Correlation Region Location and Topology

[25] Out of the 52 modeled correlation regions on 23 August 2003 using the reverse ray tracing technique, Figure 7a shows a histogram of the source frequency emission drift for the 29 events on 23 August 2003. Every bin on the histogram represents a range of 0.5 kHz/s and each of the 29 events has a range of source frequency emission drifts that may span more than one bin. The source frequency emission drift for these events ranges from 1 to 6 kHz/s, with the most common value being somewhere between 1.5 and 2 kHz/s. These rates correspond to two-thirds or more of the total slope observed on the spectrograms. The remaining variation with time is due to wave dispersion along the raypath. The source frequency emission drift was also identified for 10 events on the other 3 days and ranges from 6.8 to 20 kHz/s.
47 are found in their entirety to be earthward of the spacecraft L-shell whereas the other five span from the earthward side of the L-shell to the antiearthward side. The modeled regions have a Gaussian-like distribution of distance (meridional $z$) from the spacecraft (Figure 4b) peaked at $\sim R_E$, and are found to be close to the magnetic equator (Figure 4c). Since there are likely to be many active chorus sources in the magnetosphere at any given moment, spacecraft not within the chorus source region itself are generally exposed to the subset of sources that are at a certain distance and angular separation from the spacecraft. In addition, an individual source that is detected by the spacecraft likely emits many rays that do not reach the four spacecraft. Thus, this distance represents the best distance for viewing a single chorus emission on all four spacecraft and not the only distance at which chorus emissions from a source can be detected. It is also important to note that the overall shape of the wave normal histograms in Figure 5 is not necessarily indicative of the shape of the ray distribution emitted by an individual source, but only the rays that reach the spacecraft. However, the variability in correlation region size and location relative to the spacecraft for the 52 events on 23 August 2003 makes it likely that the entire span of wave normal angles emitted by these regions is represented in the histograms. Hence the histograms do yield some information on the distribution of wave normals in chorus sources.

[26] As noted before, the modeled correlation regions do not necessarily reflect the true topology of the sources emitting the observed waves. However, they need to have at least one common point with the real source. Figure 8 shows four correlation regions modeled in a small span of time to illustrate the variability in correlation region size and distance from the spacecraft. Both the perpendicular and parallel extent of each correlation region found from this technique vary in size from one event to another. Correlation regions $a$ and $b$ are typical in that they extend more along the magnetic field direction than perpendicular to it, while regions $c$ and $d$ show more of a perpendicular extent. In addition, region $d$ is separated from the general location of the other three correlation regions.

[27] There is an overall tendency of the modeled regions to be relatively compact in the direction perpendicular to the magnetic field and to be more extended along the field line (Figure 4a). This general shape is consistent with observations by Santolík et al. [2004], who found an average extent of $3000–5000$ km along the magnetic field and Santolík and Gurnett [2003], who found the perpendicular extent to be confined to within $\sim 100$ km of the source field line.

6.2. Wave Distribution Comparison to Published Results

[28] In order to compare these distributions with published wave normal measurements, the events on 23 August 2003 have been divided into two groups: those with wall correlation regions within $4^\circ$ of the magnetic equator, and those outside of $4^\circ$, as shown in Figure 9. Both groups are further separated into upper and lower band, once again determined from the estimated equatorial value of the cyclotron frequency on the magnetic field line of each spacecraft. Most of the wave normal angles of lower band chorus within $4^\circ$ of the magnetic equator are confined to within $20^\circ$ of the magnetic field, consistent with the source region observations of Burton and Holzer [1974], Goldstein and Tsurutani [1984], and Santolík et al. [2003]. There are however quite a few rays emitted from the source at $\sim 50^\circ$. Figure 9c shows that the spacecraft are observing more oblique rays from correlation regions outside of $4^\circ$ with a large peak at $\sim 50^\circ$. The upper band chorus shows more similarity between the two histograms with a small but noticeable shift toward smaller values of the wave normal angle for sources outside of $4^\circ$.

[29] Overall, the correlation regions are seen to emit radiation in a broad spectrum of wave normal angles. Because of the paucity of events on the other 3 days no results of this sort are presented.

6.3. Rays With Connection to Plasmaspheric Hisss

[30] Recently, Meredith et al. [2004] established a correlation between plasmaspheric hiss intensity from 0.1 to 2 kHz and substorm activity. This was followed by a ray tracing study by Chum and Santolík [2005] which showed...
that chorus, whose frequency and intensity is also related to substorm activity, can propagate into the plasmasphere where it mixes to form the incoherent hiss spectrum. A more comprehensive ray tracing study by Bortnik et al. [2008] showed, with a multitude of equatorial chorus sources at different L-values, that the range of chorus frequencies that enter the plasmasphere and survive the longest before becoming severely Landau damped is the same as corresponds to the peak power of the hiss spectrum in much of the plasmasphere. These papers provide strong evidence that the incoherent hiss spectrum is formed by chorus waves.

**Figure 8.** The correlation regions modeled for four chorus events that occur in a short span of time. The spacecraft are located at the cross at the bottom of each plot. The curved lines are the spacecraft L-shell and the L-shell of 5, and the dotted lines are latitude lines of ±1, ±3, ±5, and −10⁰. Figures 8a and 8b have a larger parallel extent than perpendicular extent, while Figures 8c and 8d have a more significant perpendicular extent. Also, Figure 8d is separated by a significant distance from the other three.

**Figure 9.** Wave normal angles θₙ (degrees) emitted by the modeled correlation regions on 23 August 2003 separated into regions within 4° of the magnetic equator and regions outside of 4°. The black and gray colors represent earthward and antiearthward wave normal angles, respectively. (a) Lower band rays within 4°, (b) upper band rays within 4°, (c) lower band rays outside of 4°, and (d) upper band rays outside of 4°.
Comparison Between Days

6.4. Source Frequency Emission Drift Variation and 
[2008].

The position of the spacecraft (almost identical for both events) at $x = 4.84 R_E$ and $z = -0.25 R_E$. The solid line is the frequency/time slope determined at the spacecraft position from the forward ray tracing simulation. This slope is due to dispersion and a source frequency emission drift with constant values of 3.1 kHz/s. The dashed line that terminates at $\sim 3.7$ kHz is the best fit slope of the chorus element in Figure 6a and is an example of an event whose source frequency emission drift is constant throughout the generation of the chorus element. The other dashed line is the best fit slope for the chorus element in Figure 6b. The deviation of this line from linearity is much more substantial than that of the solid line which indicates that the source frequency emission drift changes throughout the generation of the chorus element.

evidence that hiss at frequencies <2 kHz originates from discrete chorus emissions.

[31] The connection is less well established for frequencies >2 kHz. At these higher frequencies hiss intensity correlates well with the global distribution of lightning strikes [Green et al., 2005, 2006; Thorne et al., 2006]. However, part of the hiss spectrum in this range may also be related to chorus. In particular, Bortnik et al. [2008, Figure 3e] shows that $\sim 2$–4 kHz chorus waves emitted from the magnetic equator at an L-value corresponding to $f = 0.3 f_m$ and with wave normal angles in the range of $-30^\circ$ to $-45^\circ$ propagate into the plasmasphere with lifetimes of up to 15 s. This wave normal range is very similar to the peak at $\sim 20$ to $-30^\circ$ (and which extends out to $-50^\circ$) in Figure 5 of this paper. The results of this study therefore indicate that chorus sources do indeed emit these rays and that chorus waves, in addition to lightning whistlers contribute to the hiss spectrum in the frequency range from 2 to 4 kHz. It is important to point out that the correlation regions are located close to, but not exactly at the magnetic equator as they were in the simulations by Bortnik et al. [2008].

6.4. Source Frequency Emission Drift Variation and Comparison Between Days

[32] The source frequency emission drift rate for 29 of the 52 events on 23 August 2003 has been found to range from 1 to 6 kHz/s, with the most common rate being between 1.5 and 2 kHz/s. Figure 7b shows the minimum percentage of the total frequency/time slope accounted for by the source frequency emission drift for these events. This shows that the source frequency emission drift accounts for the majority of the frequency/time slope of a chorus element as seen on the Cluster spacecraft, while propagation dispersion accounts for the rest. Thus, for the modeled correlation regions on 23 August 2003, most of the frequency variation is created internal to the source. For the other days the source frequency emission drift is found to range from 6.8 to 20 kHz/s. The lower end of this range is a significant fraction of the total frequency/time slope while the upper end is an insignificant fraction of it. These source frequency emission drift values are in reasonable agreement with theoretical drift rates from models of chorus source regions by Trakhtengerts et al. [2004] and Omura et al. [2008] and observational values of approximately 2 kHz/s [Burris and Helliwell, 1976].

[33] In addition, it is suggested that the source frequency emission drift for the 23 events of the type shown in Figure 6b actually changes value during the generation of the chorus element whereas it has a constant value for the 29 events of the type shown in Figure 6a. A forward ray tracing simulation for the two chorus events in Figure 6 was performed from a point located at $x = 4.84 R_E$ and $z = -0.25 R_E$ in the meridional plane. This point exists within the correlation region found for both events. The location of the spacecraft changes little between the events and is taken to be at $x = 4.59 R_E$ and $z = -1.09 R_E$. The solid line in Figure 10 is the simulated frequency/time slope that results from a combination of dispersion and a constant value of source frequency emission drift taken to be 3.1 kHz/s. This line deviates only slightly from linearity because the higher frequencies disperse more than the lower frequencies and thus arrive at the spacecraft at a slightly later time. The dashed line that terminates at $\sim 3.7$ kHz in Figure 10 is the actual chorus slope for the event in Figure 6a. This line is essentially identical to the solid line which indicates that the source frequency emission drift has a constant value throughout the generation of this event. The other dashed line, which represents the actual slope of the chorus event in Figure 6b, deviates substantially from the simulated line. This suggests that the source frequency emission drift changes throughout the generation of this event. Since the simulations for the two chorus events were identical in every respect except for the different frequency ranges it is likely that the change of the source frequency emission drift for the second event is due to the larger frequency range. If the event in Figure 6a had a maximum frequency corresponding to the event in Figure 6b then it too might display a similar changing value of the source frequency emission drift. It is worth mentioning that it is possible in some cases that the observed change in slope could be due to the overlapping of two different chorus elements generated at different frequency bandwidths. Also, because the frequency variation from propagation dispersion over the frequency ranges and ray distances simulated is essentially linear, our approach of simulating only the maximum and minimum frequency for each event and neglecting the intermediate frequencies is justified.

[34] These results suggest that any realistic model of chorus emission must include a source frequency emission
but other angles appear as well. In the higher of the two other angles. many rays near the Gendrin angle but also many rays at many rays near the Gendrin angle, while the upper one has many rays near the Gendrin angle but also many rays at other angles.

drift. Because events were found on only 4 separate days, it is not statistically meaningful to compare the simulated source frequency emission drift with any parameter in the density or magnetic field model.

6.5. Waves Near the Gendrin Angle

Studies by Goldstein and Tsurutani [1984] and Lauben et al. [2002] have indicated that chorus rays in the source region can be concentrated at the Gendrin angle. Waves at the Gendrin angle propagate with a group velocity parallel to the magnetic field and are thought to stay in resonance with counterstreaming electron beams near the magnetic equator over a longer distance than waves at other nonzero wave normal angles. In order to highlight the importance of the Gendrin angle, Figure 11 shows the distribution of the quantity $|\theta_k - \theta_{\text{Gendrin}}|$ for the frequency ranges $f_{\text{fc}} = 0.3 - 0.4$ and $f_{\text{fc}} = 0.4 - 0.5$ for the 23 August 2003 events. The local cyclotron frequency in this case is the local cyclotron frequency at every point within the correlation region. A value of zero indicates propagation within five degrees of the Gendrin angle. No upper band values are included because the Gendrin angle is always zero for the upper band. Waves within five degrees of the Gendrin angle are the most common in the frequency band $f_{\text{fc}} = 0.3 - 0.4$, but other angles appear as well. In the higher of the two frequency bands ($f_{\text{fc}} = 0.4 - 0.5$) there are still many rays near the Gendrin angle, but also many rays with wave normals significantly different than the Gendrin angle.

The frequency range from $f_{\text{fc}} = 0.3$ to 0.4 (Figure 11a) corresponds to Gendrin angles of $\pm 37 - 53^\circ$. A comparison of this range to the wave normal histograms in Figure 5 must be made with care since the wave normal histograms are normalized to the equatorial value of the cyclotron frequency. The local cyclotron frequency at a given $L$-value of any source not at the equator will always be higher than the equatorial value in a dipole magnetic field. Therefore the lower band wave normal histogram will consist of a blend of all the lower band waves in Figure 5a and some of the upper band waves in Figure 5b which are now “locally” lower band. This blended histogram has a large peak at positive wave normal angles of $\sim 20 - 50^\circ$ as well as many rays at negative wave normal angles within the stated range of Gendrin angles.

These histograms suggest that the Gendrin angle may play an important role in the generation of chorus emissions. Rays near the Gendrin angle are more prevalent for lower frequencies and become less important as the frequency approaches $1/2 f_{\text{c}}$. Waves near the Gendrin angle do not play an exclusive role, however, and other wave normal angles are seen to be emitted by the correlation regions. This is in agreement with Chum et al. [2007], whose ray tracing study suggests that a broad spectrum of wave normal angles is emitted by chorus sources.

7. Conclusions

The frequency/time characteristics of chorus emissions as seen on the Cluster Wide Band Data instrument on multiple Cluster spacecraft are simulated with a ray tracing technique to identify possible chorus source locations and emission characteristics. The correlation regions are, for the most part, found to be earthward of the spacecraft $L$-shells, within a few degrees of the magnetic equator and are extended more along the Earth’s magnetic field direction than perpendicular to it. These correlation regions emit both lower and upper band rays in a wide spectrum of wave normal angles. This distribution of wave normal angles observed at the spacecraft depends on the proximity of the correlation region to the magnetic equator. Correlation regions close to the magnetic equator emit lower band rays with wave normal angles generally confined to within $20^\circ$ of the magnetic field while upper band rays are generally found at higher wave normal angles of between $-20$ and $-40^\circ$, and at $50^\circ$. The peak at $50^\circ$ is due to the combined contribution from a number of correlation regions similar to the one presented in Figure 4a. As the rays in both bands propagate to higher latitudes the average value of the wave normal angle increases.

Of particular interest is the peak in the lower band wave normal histogram (Figure 5) at $\sim 20$ to $30^\circ$ that shows that the identified correlation regions preferentially emit the rays that may be the progenitors to the plasmaspheric hiss spectrum from $\sim 2$ to 4 kHz. Thus it seems likely that chorus in addition to lightning whistlers contribute to the hiss spectrum in this range.

Also, many $f < 1/2 f_{\text{c,local}}$ rays in the source at low frequencies are near the Gendrin angle, although this is by no means exclusive. As the frequency increases fewer rays propagate near the Gendrin angle. One consequence of the propensity of rays to be at small to moderate wave normal angles, or, in the case of $f < 1/2 f_{\text{c,local}}$ rays, near the Gendrin angle, is that rays of both bands propagate almost exclusively along the magnetic field direction.

The source frequency emission drift has been modeled for 29 of the 52 correlation regions on 23 August 2003. For these events the source frequency emission drift appears to have a constant value in time throughout the generation of the chorus event and ranges from 1 to 6 kHz/s. This rate typically accounts for 2/3 or more of the frequency/time slope as seen on the spectrograms, the rest being due to frequency separation by propagation dispersion. The source frequency emission drift for events on the other 3 days ranges from 6.8 to 20 kHz/s which ranges from a significant
fraction of the total slope on the lower end to a small fraction of it on the upper end. It has also been found that the source frequency emission drift rate may change with time during the generation of some chorus events. 

[42] The results of this paper show that any theoretical chorus source in the inner magnetosphere must emit radiation predominantly along the magnetic field direction. In other words, the source must emit rays with a range of low to moderate wave normal angles, perhaps with some emphasis for rays in the lower band to be at the Gendrin angle. Furthermore, any realistic chorus source must emit a frequency that changes as a function of time with a rate that potentially accounts for a majority of the frequency/time slope for a chorus emission at any observation point within ~20° of the source region.

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