Acoustic-gravity waves during solar eclipses: detection and characterization using wavelet transforms.

P. Šauli(1), S. G. Roux(2), P. Abry(2), J. Boška(1)

(1) Institute of Atmospheric Physics ASCR, Czech Republic, pkn@ufa.cas.cz, boska@ufa.cas.cz
(2) Laboratoire de Physique (UMR5672), CNRS, Ecole Normale Supérieure, Lyon, France, Patrice.Abry@ens-lyon.fr, perso.ens-lyon.fr/patrice.abry

Abstract

In the present contribution, we first propose a methodology that enables to detect wave like structures propagating in Ionosphere, by tracking the local maxima of the modulus of continuous complex wavelet transform coefficients with respect to heights. From the derivation of the phases of the wavelet transform, we measure the corresponding propagation parameters. These tools are applied to measurements collected by vertical ionospheric sounding at high-time resolution sampling regime (sampling periods ranged from 1 to 3 min) in the observatory Průhonice (49.9N, 14.5E, Czech Republic). The aim of these experiments is to analyze the changes in the ionospheric plasma induced by three different solar eclipse events (total solar eclipses, 11 August 1999, 29 March 2006, and annular solar eclipse, 3 October 2005) and to detect and analyze the propagation of the generated acoustic-gravity waves (AGW). Second, injecting wave vector components measured from the data into the AGW propagation equations, we obtain a full description of the propagation of the waves. This enables us to differentiate acoustic-gravity waves from others wave-like oscillations and to discuss similarities and differences of the waves detected during these three particular events. These procedures also enabled us to detect acoustic waves. We believe that the methodology proposed here brings significant improvement in detecting and characterizing acoustic-gravity wave propagations from empirical data and can be readily used in the Ionosphere community.

Key words: Acoustic-Gravity Wave, Vertical Ionospheric Sounding, F-Layer, Wavelet Transform, Wave Packet Characterization

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1 Introduction

Acoustic-Gravity Waves in Atmosphere. Terrestrial Atmosphere shows a high variability over a broad range of periodicities, which mostly consists of wave-like perturbations characterized by various spatial and temporal scales. Amongst atmospheric waves, acoustic-gravity waves, whose periodicities range from minutes (pure acoustic waves) to a few hours (upper limit of gravity waves), constitute the source of most of the short-time ionospheric variability. Acoustic-gravity waves play an important role in the dynamics and energetics of Atmosphere and Ionosphere. For instance, they are responsible for momentum and energy transfers from high latitudes to low latitudes and from lower to upper Atmosphere. Because acoustic-gravity waves spread energy between atmospheric regions, they significantly contribute to the global circulation, temperature and compositional structure of Mesosphere, Thermosphere and Ionosphere. AGWs are hence an important component of the atmospheric motion field. Acoustic-gravity wave propagation also have a significant impact on radio wave propagation conditions. Therefore, analyzing and understanding wave generation mechanisms with respect to specified sources constitute a major goal to improve our knowledge of atmospheric dynamics.

Acoustic-gravity wave theory (e.g. Hines 1960) in terrestrial Atmosphere has been developed in the sixties and then further extended by various authors (e.g. Hooke 1968, Hocke and Schlegel 1996, Fritts 1989). The first experimental observations refer to acoustic-gravity waves generated by explosive sources and earthquakes (cf. Harkrider 1964, Row 1967), and acoustic-gravity waves in Earth Atmosphere have been widely studied empirically since. The interests of scientists range from the analysis of the climatology of the gravity waves to case studies of acoustic-gravity wave occurrences related to particular events acting as wave sources, such as meteorological systems (see, for instance, Boška and Šaulí 2001, Kelley 1997, Martinis and Manzano 1999, Šaulí and Boška 2001), geomagnetic storm (cf. Hocke and Schlegel 1996, Bruinsma et al. 2006, Hawlitschka 2006), solar eclipse, etc. Various radio techniques (including ionosonde) were used around the globe to analyze the climatology and case events of acoustic-gravity waves. Numerous measurements and campaigns were conducted aiming at relating the observed gravity waves to their sources (e.g., HIRAC campaign amongst others, cf. Feltens et al. 2001). However, the description, interpretation and understanding of the mechanisms underlying AGW generation and propagation still remain incomplete. This is mostly due to severe difficulties in analyzing real measurements. For instance, it is difficult to decide whether the observed wave characteristics are due to the properties and positions of the AGW sources or to interactions between the propagating waves and the mean flow (convection, tides, planetary waves etc.), see e.g.,
Acoustic-Gravity Waves and Solar Eclipses. During a solar eclipse, Atmosphere strongly reacts to the break of ionization flux and heating. At thermospheric heights, the reduction in temperature causes a decrease of pressure over the totality footprint to which the neutral winds respond. Thermal cooling and downward transport of gases lead to neutral composition changes in Thermosphere that has significant influence on the resulting electron density distribution. Temperature fluctuations and electron density changes propagate as a wave, away from the totality path, cf. Muller-Wodarg et al. 1998. It has been proposed by Chimonas and Hines 1970 that solar eclipses can act as sources for acoustic-gravity waves. The lunar shadow creates a cool spot in the Atmosphere that sweeps at supersonic speed across the Earth. The sharp border between sunlit and eclipsed regions, defined by strong gradients in temperature and ionization flux, moves throughout Atmosphere and drives it into a non-equilibrium state. Earth Atmosphere shows variable sensitivity to the break of ionization flux. Studies by Fritts and Luo 1993 suggest that perturbations generated by the eclipse induced ozone heating interruption may propagate upwards into the Thermosphere-Ionosphere system where they have an important influence. At a theoretical level, Kato et al. 1977 demonstrated that, in the Atmosphere, gravity waves can easily be radiated in association with almost any motion of the source whilst acoustic waves can be emitted only by supersonic motion. By means of vertical ionospheric sounding, Liu et al. 1998 detected waves excited during Solar eclipse event at F1 layer heights and attributed their generation and/or enhancement to changes of temperatures, and variations of the height of the transition level for the loss coefficient and the height of the peak of electron production. Many different mechanisms are likely to contribute to wave generation and enhancement at Ionospheric heights. Hence, it is difficult to clearly separate or differentiate each contributing agent and to decide which part of wave field belongs to the in-situ generated and which part comes from distant regions. First experimental evidence of the existence of gravity waves in Ionosphere during solar eclipse were reported in Walker et al. 1991, where waves with periods of 30-33 min were observed on ionosonde sounding virtual heights. However, inconclusive results of the solar eclipse observations rise from the fact that different solar eclipses produce different plasma motions. Indeed, the travel cone geometry and its angular effects on the magnetized plasmas are different for each eclipse. Studies reported in Farges et al. 2001 suggest a longitudinal diversity of the disturbances with respect to pre-noon and post-noon phases.

Goals of the present contribution. Various experimental studies of the 11 August 1999 solar eclipse (cf. e.g., Farges et al. 2001 Altadill et al. 2001a,
Altadill et al. 2001b, Šauli et al. 2006b, Šauli et al. 2006a) analyzed the relations between acoustic-gravity wave generation/enhancement in Ionosphere and solar eclipse events. The present contribution aims at enlarging the scope of previously existing studies and at bringing new information about horizontal and vertical propagation characteristics. To do so, acoustic-gravity waves detected during three different solar eclipses (11 August 1999, 3 October 2005 and 29 March 2006) are studied and compared. Description of these eclipses and corresponding data are detailed in Section 2 and Table 1.

Elaborating on tools proposed in Liu et al. 1998, Altadill et al. 2001a and Šauli et al. 2006a, this contribution also develops a wavelet transform based methodology to detect wave packets (or structures) propagating at Ionospheric heights and to measure, from data, their time, period and height locations, their wave vectors, phase and packet velocities. Techniques based on Fourier transforms were previously proposed. However, by definition, Fourier transforms are averaging, and hence mixing information, along time. Therefore, wave parameters measured at a given frequency can potentially result from the contribution of different waves sharing the same characteristic frequency but existing at different time positions, hence producing poor or inaccurate characterization of the waves. Wavelet decompositions, thanks to their being joint time and frequency representations, enable to disentangle the contribution of different structures whose time or frequency supports partially overlap. Therefore, they enable to better identify and analyze wave structures, to more accurately decide whether they consist of acoustic-gravity waves or not and finally to better measure their corresponding propagation parameters.

Further developing this wavelet based approach, we inject measurements into the equations governing acoustic-gravity wave propagation and, making use of an upper Atmosphere model (Pietrobon 2000), we fully characterize the propagating parameters of the detected structures. Acoustic-gravity wave theory and the upper Atmosphere model are described in Section 3. Wavelet decomposition and wave packet detection and characterization are detailed in Section 4. In Section 5, we discuss the obtained results and conclude.

2 Solar Eclipses and Data

Solar eclipse events. In the present contribution, we analyze three different solar eclipse events. Two of them, 11 August 1999 and 29 March 2006, represent total solar eclipses, while the third one, 3 October 2005, is an annular solar eclipse. All three solar eclipse events occurred during periods of low geomagnetic activity. Indeed, as shown in Fig. 1, Kp indices remained below or equal a maximum value of 3 (out of 9) for several consecutive days, indicating that Ionosphere remained in a quiet state. Therefore, we can consider that measurements reflect the variability due to the occurrence of the solar
eclipses. The supersonic motion of the totality/annularity footprint causes shock wave structures in Atmosphere, that are further reflected in ionospheric plasma. The parameters describing each solar eclipse are given in the Table 1, as observed from Průhonice ionospheric station.

Data Measurements and Time Series. In upper Atmosphere, acoustic-gravity waves are observed directly as fluctuations of neutral gas or oscillations of the ionospheric plasma due to the coupling between the neutral and ionized components. Our measurements consist of vertical profiles of electron concentration. For the three eclipses, measurements were performed at the European mid-latitude ionospheric station Průhonice (Czech Republic; 49.9N, 14.6E), using vertical ionospheric sounding techniques. The 1999 solar eclipse was monitored using a classical ionosonde IPS 42 Kel Aerospace and data were collected with a 1-minute sampling period. This ionosonde was later (January 2004) replaced by the Digital Portable Sounder 4 (DPS4) hence used for the two latest solar eclipse events. For DPS4 measurements, lower resolution regimes were chosen to enable simultaneous record of ionograms and plasma drift. Hence data were collected with 2-minute and 3-minute sampling periods for the 2005 and 2006 events, respectively.

Real height vertical electron density profiles were derived from ionograms using two inversion techniques POLynomial ANalysis (Titheridge 1985) and NHPC (Huang and Reinish 1996). Finally, from the real height electron density profiles, we obtain variations of the electron concentration $X$, as a function of time $t$, at fixed heights $z$:

$$X(t, z), \ t \in [T_m, T_M], z \in Z.$$  \hspace{1cm} (1)

where $T_m$ and $T_M$ denote the beginning and end of the measurement in UT. The spatial sampling period is 5km, corresponding to heights $Z = \{155, 160, 165, \ldots, 255\}$ (in km). The time series for the three eclipses are shown in Fig. 2, left column. The acoustic-gravity wave detection procedures described below are performed on these $X(t, z)$ time series.

3 Acoustic-Gravity Wave theory

Acoustic-Gravity Wave Propagation At periods of minutes and larger, buoyancy effects become important due to atmospheric stratification and Atmosphere becomes dispersive and anisotropic. In such a medium, phase and energy no longer propagate along the same direction. Under an energy conservation assumption, the propagation of acoustic-gravity waves is driven by the following ideal dispersion relation:

$$\omega^4 - \omega^2 \omega_g^2 - k_z^2 C^2 (\omega^2 - \omega_g^2) - C^2 \omega^2 k_z^2 = 0,$$  \hspace{1cm} (2)
where \( k_x \) and \( k_z \) stand for the horizontal and vertical components of the wave vector, \( C \) for the speed of sound, \( \omega_a \) for the angular acoustic cut-off frequency and \( \omega_g \) for the angular buoyancy (or Brunt-Väisälä) frequency. This dispersion relation accounts for a nonlinear and dispersive propagation. It indicates the existence of two propagation frequency ranges: acoustic modes, with characteristic frequencies larger than the acoustic cut-off \( \omega_a \), gravity modes, with characteristic frequencies smaller than the Brunt-Väisälä \( \omega_g \). An important property of the gravity mode consists of that fact that energy flows up when phase travels down and vice versa, while for acoustic mode both energy and phase propagate jointly, either upward or downward. The phase propagation angle \( \Phi \) (measured from the vertical, clock-wise) indicates the phase velocity (or wave vector) direction while the energy propagation angle \( \gamma \) (measured from the wave vector direction, clock-wise) indicates the packet velocity direction:

\[
\tan \Phi = \frac{k_x}{k_z},
\]

\[
\tan \gamma = \left( \frac{\omega_a}{\omega} \right)^2 \sin \Phi \cos \Phi \bigg) / \left( 1 - \left( \frac{\omega_a}{\omega} \right)^2 \sin^2 \Phi \right).
\]

The modulus of the wave vector, the phase velocity and the vertical and horizontal components of the packet velocity are defined as:

\[
k = k_x^2 + k_z^2,
\]

\[
v_\phi = \frac{\omega}{k}.
\]

\[
v_{p,x} = \left( C^2 k_x (\omega^2 - \omega_a^2) \right) / \left( \omega (2 \omega^2 - \omega_a^2 - C^2 k^2) \right),
\]

\[
v_{p,z} = \left( C^2 k_z \omega^2 \right) / \left( \omega (2 \omega^2 - \omega_a^2 - C^2 k^2) \right).
\]

Eqs. (2) to (8) are derived in e.g., Hines 1960 or Davies 1990.

**Neutral Atmosphere Parameters.** For a practical use of Eqs. (2) to (8), it is necessary to set the values of \( \omega_a \), \( \omega_g \) and \( C \) that reflect the properties of the background neutral Atmosphere. In our analysis, we consider that the upper Atmosphere is well described by the Australian Standard Atmosphere model 2000 (UASA2000). The UASA2000 model is based on U.S. Standard Atmosphere, 1976 (USSA1976), and has been modified in the upper Atmosphere, above 86km (Pietrobon 2000). The UASA2000 model provides the scale height and acceleration due to gravity that are necessary to compute the speed of sound. The ratio between the specific heats at constant pressure and constant volume is a key factor in adiabatic processes and in determining the speed of sound in a gas. This ratio takes the value \( \gamma = 1.66 \) for an ideal mono-atomic gas and \( \gamma = 1.4 \) for a diatomic gas. Because Earth Atmosphere is predominantly a diatomic gas, we use this latter approximation (Davies 1990, Pröls 2004).
4 Acoustic-gravity wave detection and Characterization

4.1 Time-Scale wave packet decomposition

The goals of the analyses of the data $X(z,t)$ are to detect the occurrence of coherent wave packets as well as to extract the corresponding wave propagation information. In the literature, this has traditionally been addressed by performing a wave packet expansion of the data by means of Fourier Transform. A methodology, originally introduced in Liu et al. 1998, Altadill et al. 2001a, proposed to detect waves from the tracking of the maxima of the modulus of the Fourier transforms of the data at different heights $z$. Propagation parameters were then computed from the corresponding phases of these Fourier transforms. Elaborating on a previous contribution (cf. Šauli et al. 2006a), we extend this original idea to the use of a joint time and frequency representation of the data: the complex valued continuous wavelet transform. Instead of Fourier coefficients, we compute wavelet coefficients, $T_X(w,t,z)$, that account for the information contained in the data $X(t,z)$, at height $z$, jointly around the time position $t$ and the period $P = 2\pi/w$.

Such time-period coefficients are obtained by comparisons, by means of inner product, of the data $X(t,z)$ against a family of analyzing functions: $T_X(w,t,z) = \langle X(\cdot, z), 1/\sqrt{a}\psi_0((\cdot - t)/a) \rangle$, where $\psi_0$ is the so-called mother wavelet, $a$ the analysis scale, such that $P = 2\pi a/w_\psi$, ($w_\psi$ is a constant pulsation characterizing $\psi_0$, cf. Šauli et al. 2006a). For a thorough introduction to wavelet transforms, the reader is referred e.g., to Mallat 1998. In the present work, we adapted the wavelet decomposition Matlab toolbox provided by Torrence and Compo (cf. Torrence and Compo 1998) to our purposes.

The key ingredient of the wavelet analysis lies in the fact that it constitutes a joint time-Period representation of the data so that the use, as a mother wavelet, of any oscillating pattern with a satisfactory joint time and frequency localization essentially yields comparable results. In the present contribution, for sake of simplicity, we make use of a single function $\psi_0$, the Paul mother-wavelet, defined as:

\[
\text{Paul : } \psi_{0,N}(t) = \frac{2^N N!}{\pi (2N)!} (1 - it)^{-(N+1)},
\]

where $N$ provides a degree of freedom, mostly controlling the number of oscillations of the mother-wavelet, that can be easily tuned to a given purpose. Parameter $N$ is set to 4, as the corresponding $\psi_0$ exhibits satisfactory joint time-period resolutions. Paul 4 wavelets $\psi_a(t) = 1/\sqrt{a}\psi_0(t/a)$ (and their Fourier transforms) are represented in Fig. 3 for two different scales $a$, corresponding respectively to characteristic periods $P = 5\text{min}$ and $P = 30\text{min}$. AGW mostly corresponds to waves with periods ranging from a few minutes.
to an hour. The range of analysis scales $a$ is varied so that the characteristic periods of the wavelets $\psi_a(t)$ covers this entire range.

However, choosing specific mother wavelets $\psi_0$ could help to better match specific waveforms in the data and hence theoretically improve the wave-packet detection and characterization. For instance, the celebrated Morlet wavelet (with standard parameters) benefits from a better period localization for large periods, compared to that obtained with Paul 4 wavelet. Instead, the time resolution of Paul 4 wavelet is better. Therefore, a practical rule may be to use Paul wavelet when aiming at detecting structures which are strongly localized in time with low periods, while Morlet wavelet may be preferred for structures with much larger characteristic periods and hence more spread in time. Acoustic waves, with periods of the order of the minutes, are very concentrated in time. Therefore, their detection and characterization is better achieved using mother wavelets with good time resolution. For instance, in Figs. 12 and 13, showing results for an acoustic wave, we choose to use a Paul mother wavelet, with parameter $N = 2$ to further increase the time resolution.

### 4.2 Wave-packet detection

The structure detection scheme that we propose is organized in three key major steps:

1. Data preprocessing and wavelet decomposition,
2. Energy concentration detection at each altitude and maxima line tracking along altitude,
3. Wave parameter measurements.

They are detailed below. For explanation purposes, the behavior of the entire detection and characterization procedures is illustrated on a specific example structure, (corresponding to a superb gravity wave) occurring during the August, 11th, 1999 eclipse (GW1, in Table 2). The corresponding data are shown in Fig. 2 (left plots).

#### 4.2.1 Data preprocessing and wavelet decomposition

For each altitude $z$ independently, a high-pass filter is applied to the time series $\{X(t, z), t \in [T_m, T_M]\}_{z \in \mathbb{Z}}$ to suppress periods larger than 90 min and focus on short term oscillations. Detrended time series are shown Fig. 2 (right plots). Then, complex wavelet coefficients are computed on these detrended data. Examples of scalograms are presented in Figs. 4 and 11.
4.2.2 Wave packet detection

First, for the scalograms $|T_X(\omega, t, z)|$, at each $z$ independently, local energy maxima are detected and their time position, period, amplitude and phase recorded. Second, local maxima that exist jointly over a continuous range of heights $z$, within a same time-period neighborhood are connected together to form maxima lines. When different maxima exist in a same time-period neighborhood, the chaining operation is conducted to favor smooth evolutions along $z$ of the local maxima parameters. Each of these maxima lines correspond to the detection of a wave packet (or wave structure), and consists of the following collection of information:

(1) Altitude range $z \in [\bar{z}, \overline{z}]$ within which the structure is detected;
(2) Precise time position $t_0(z)$ and pulsation $\omega_0(z)$ of the occurrence of the maximum at each height $z$ and the corresponding amplitude $X_0(z) = X(t_0(z), z)$;
(3) Modulus $|T_X(\omega, t, z)|$ and phase $\phi(\omega, t, z)$ of the wavelet coefficients in the time-period neighborhood around the maxima position.

Scalograms, corresponding to different altitudes, showing local maxima marked with ‘•’ are displayed in Fig. 4(a),(c) and (e). The practitioner can make use of a set of tools for visual inspection of the scalograms and for the manual selection of the structure (or maxima line) he wants to analyze. Then, he can zoom in the scalograms around the time and period locations of the chosen structure. This is illustrated in Fig. 4(b),(d) and (f) for the same altitude. The selected structure is marked by the use of a ‘▲’.

4.2.3 Wave packet measurement

From the information collected for each wave packet, we derive the following collection of attributes.

(1) Mean time and pulsation defined as:

$$t_0 = \langle t_0(z) \rangle_z, \quad \omega_0 = \langle \omega_0(z) \rangle_z,$$

where $\langle \cdot \rangle_z$ means that average is taken over the range of altitude $z \in [\bar{z}, \overline{z}]$.

(2) Components of the Wave vector, phase and packet velocities are measured as:

$$k_z(\omega, t, z) = \partial \phi(\omega, t, z)/\partial z, \quad k_{0,z}(z) = \langle \langle k(\omega, t, z) \rangle \rangle_{t_0(z), \omega_0(z)},$$

$$v_\phi^{(z)}(\omega, t, z) = \omega/k_z(\omega, t, z), \quad v_{\phi,0}^{(z)}(z) = \langle \langle v_\phi^{(z)}(\omega, t, z) \rangle \rangle_{t_0(z), \omega_0(z)},$$

$$v_p,z(\omega, t, z) = \partial \omega/\partial k_z(\omega, t, z), \quad v_{p,0,z}(z) = \langle \langle v_p,z(\omega, t, z) \rangle \rangle_{t_0(z), \omega_0(z)};$$

where $\langle \langle \cdot \rangle \rangle_{t_0(z), \omega_0(z)}$ denote that we take the median within a narrow time-
pulsation neighborhood centered around \( t_0(z) \) and \( \omega_0(z) \).

Fig. 5 shows \( t_0(z), P_0(z) = 2\pi/\omega_0(z), k_{0,z}(z), X_0(z), v_{\phi,0}(z) \) and \( v_{p,0,z}(z) \) measured according to the procedure described above for the chosen example structure.

This theoretically simple procedure calls for three important practical comments.

**Comment 1** Computing the quantities above involves derivation. This is performed using a third-order or fifth-order finite difference procedure, depending on the range of heights available in the structure and border effects are taken care of. Note that the computation of \( v_{p,z}(\omega,t,z) \) requires a double derivation and is actually computed as the inverse of \( \partial/\partial \omega(\partial \phi(\omega,t,z)/\partial z) \). This is numerically poorly conditioned and may lead to inaccurate results. This is further discussed in Section 5.3 where an acoustic wave is analyzed.

**Comment 2** From ionospheric vertical sounding measurements, one only has access to vertical profiles of electron density and hence to the vertical components of the wave vector, phase and packet velocities. Therefore, in Eq. (11) above \( k_z \) and \( v_{p,z} \) stand for the vertical components of the corresponding vectors. For the phase velocity, the situation is even more involved. In Liu et al. 1998, Altadill et al. 2001a, Šauli et al. 2006a, \( v_{\phi}^{(z)} \) has been incorrectly associated to the \( z \) component of the phase velocity \( v_{\phi,z} \). However, \( v_{\phi} = \omega/k = \omega/k_z \cdot k_z/k = v_{\phi}^{(z)} \cos \Phi \), while \( v_{\phi,z} = v_{\phi} \cos \Phi \), hence, \( v_{\phi,z} = v_{\phi}^{(z)} \cos^2 \Phi \). When the phase propagation direction is close to the vertical direction, the error is negligible, this is however not the case for close to horizontal phase propagation.

**Comment 3** The key feature of this procedure consists of the fact that all quantities are computed for each triplet \((\omega,t,z)\) independently and a priori. A local averaging (for instance using a median filter) is performed a posteriori over a narrow time-pulsation neighborhood for each quantity individually. The alternative choice which would consist of computing Eqs. (11) directly from averaged quantities, such as \( k_{0,z}(z) \) would lead to much poorer results.

4.3 Wave-packet characterization

No further information can be extracted from the data themselves. To decide whether a detected wave packet corresponds or not to the propagation of an acoustic-gravity wave, it can be compared to the theoretical acoustic-gravity wave propagation model recalled in Section 3, making use of the upper Atmo-
sphere model.

First, from the Atmosphere model, we derive the values of \( \omega_a(z), \omega_g(z) \) and \( C(z) \), for all \( z \in [z, z] \). Comparing the measured \( v_0(z) \) to \( \omega_a(z) \) and \( \omega_g(z) \) enables to check whether the detected structure packet consists of a gravity or acoustic waves.

Second, for all \( z \in [z, z] \) and all \( t \) and \( \omega \) in the time-period neighborhood associated to the studied structure, we derive \( k_x(\omega, t, z) \) from the dispersion relation in Eq. (2) by plugging-in the measured \( k_z(\omega, t, z) \) and the calculated \( \omega_a(z), \omega_g(z) \) and \( C(z) \).

Third, making use of Eqs. (3) to (4), we derive the phase and energy propagation angles \( \Phi(\omega, t, z) \) and \( \alpha(\omega, t, z) = \Phi(\omega, t, z) + \gamma(\omega, t, z) \), respectively, measured clockwise from the vertical direction. Then, we compute the wave vector \( k(\omega, t, z) \) and the phase velocity \( v_\phi(\omega, t, z) \) from Eqs. (5) to (6). Combining previous results yields the phase velocity components \( v_{\phi,x}(\omega, t, z) = v_\phi(\omega, t, z) \cos \Phi(\omega, t, z), \quad v_{\phi,z}(\omega, t, z) = v_\phi(\omega, t, z) \sin \Phi(\omega, t, z) \). To finish with, Eqs. (7) to (8) provide the packet velocity components \( v_{p,x}(\omega, t, z) \) and \( v_{p,z}(\omega, t, z) \).

Fourth, from these quantities, we compute the median \( \langle \cdot \rangle t_{0(z),\omega_0(z)} \) (as defined in Section 4.2 above) for each quantity hence obtaining

\[
\begin{align*}
\text{Wave Vector} & \quad k_{0,x}(z) = \langle \langle k_x(\omega, t, x) \rangle \rangle t_{0(z),\omega_0(z)} , \\
& \quad k_0(z) = \langle \langle \sqrt{k_x(\omega, t, x)^2 + k_z(\omega, t, z)^2} \rangle \rangle t_{0(z),\omega_0(z)} , \\
\text{Wavelength} & \quad \lambda_0(z) = 2\pi/k_0(z) , \\
\text{Phase Angle} & \quad \Phi_0(z) = \langle \langle \Phi(\omega, t, z) \rangle \rangle t_{0(z),\omega_0(z)} , \\
\text{Energy Angle} & \quad \alpha_0(z) = \langle \langle \alpha(\omega, t, z) \rangle \rangle t_{0(z),\omega_0(z)} , \\
\text{Phase velocity} & \quad v_{\phi,0,x}(z) = \langle \langle v_{\phi,x}(\omega, t, z) \rangle \rangle t_{0(z),\omega_0(z)} , \\
& \quad v_{\phi,0,z}(z) = \langle \langle v_{\phi,z}(\omega, t, z) \rangle \rangle t_{0(z),\omega_0(z)} , \\
& \quad v_{\phi,0,x}(z) = \langle \langle v_{\phi,z}(\omega, t, z) \rangle \rangle t_{0(z),\omega_0(z)} , \\
\text{Packet velocity} & \quad v_{p,0,x}(z) = \langle \langle v_{p,x}(\omega, t, z) \rangle \rangle t_{0(z),\omega_0(z)} , \\
& \quad v_{p,0,z}(z) = \langle \langle v_{p,z}(\omega, t, z) \rangle \rangle t_{0(z),\omega_0(z)} , \\
& \quad v_{p,0,x}(z) = \langle \langle \sqrt{v_{p,x}(\omega, t, z)^2 + v_{p,z}(\omega, t, z)^2} \rangle \rangle t_{0(z),\omega_0(z)} ,
\end{align*}
\]

Again, and as in the detection step, the central point of this characteriza-
tion step lies in all quantities being computed for each triplet \((\omega, t, z)\) independently followed by a local median, performed \textit{a posteriori} over a narrow time-pulsation neighborhood.

Moreover, it is a remarkable fact that all the calculations related to the wave characterization require the use of a single quantity measured from the data: \(k_z(\omega, t, z) = \partial \phi(\omega, t, z)/\partial z\) and enables comparisons against other measured quantities such as the \(z\)-components of the packet and phase velocities. Fig. 6 shows, for the chosen example structure, the quantities computed according to the procedure described above. Also, it compares the computed \(v_{\phi,0,z}(z)/\cos^2\Phi_0(z)\) with the measured \(v_{\phi,0}^{(z)}\) as well as the computed and measured \(z\)-component of the packet velocities.

4.4 Wavelet based acoustic-gravity wave detection and characterization toolbox

All procedures and programs used to detect and characterize acoustic-gravity wave were written and implemented in MATLAB, by ourselves. A graphical user interface of this toolbox is implemented for a friendly use and allows an easy selection of the desired structure. This toolbox is fully operational and available upon request.

5 Results and discussions

It is rather uneasy to unambiguously assess causality between the solar eclipse events and the detected wave structures in the ionospheric plasma. Difficulties result from the fact that there are no two exactly identical solar eclipse events and from limitations of sounding techniques. Despite the fact that various AGW sources have been identified, many others remain to be found. Amongst irregular AGW bursts, regular increases of AGW activity were found to occur around sunrise and sunset hours, excited by Solar Terminator movement (cf. e.g., Somsikov 1991 or Šáuli et al. 2006b). Most of other sources (meteorological systems, geomagnetic and solar disturbances, etc.) and corresponding wave-like oscillations contribute to the irregular patterns of AGW activity observed in the Ionospheric plasma. Because the solar eclipses analyzed in the present contribution occur sufficiently after the sunrise hours, we can assume that none of the reported waves are induced by Solar Terminator. During the analyzed sounding campaigns, no wave coming from auroral zone was expected, due to the quiet geomagnetic and solar activity. Additionally, analysis of meteorological situation above Europe reveals that the Průhonice observatory was in the flank of high pressure ridge. Only rests of dissipating frontal
systems were observed. Hence, there is a very low probability of influence, on the Ionosphere, of AGW launched from meteorological systems during studied events.

5.1 Wave activity

The three solar eclipses are characterized by an increase of the wave-like oscillation in the acoustic-gravity period range during and after the event. This finding is in agreement with other experimental studies related to observation of the August 11, 1999 event (see, for instance, Altadill et al. 2001a, Altadill et al. 2001b, Farges et al. 2003). However, the amplitudes of the oscillations do not remain at the same level during the whole solar eclipse event. Fig. 2(b) shows much larger amplitudes of the fluctuations during the initial phase compared to those occurring after the maximum solar disk occultation. Figs. 2(d) and 2(f) indicate completely different situations: Larger electron concentration oscillation amplitudes are observed after the eclipse maximum and remain present after the fourth contact. Moreover, the two most recent eclipses are characterized by significantly lower magnitudes compared to that of the first one. The decrease of the solar radiation flux is proportional to the magnitude of the eclipse and is reflected in the depletion of the electron concentrations at all ionospheric heights (compare Fig. 2, left column, plot (a) against (c) and (e)).

After removal of the global decrease/increase, the residual oscillations are analyzed using the wave detection procedures described above. This reveals that numerous wave-packets are detected propagating within the Ionospheric plasma, before, during and after the solar eclipse events. Most observed waves are characterized by periods ranging from 20 minutes to 70 minutes and all of them but one consist of gravity waves. All the detected and analyzed waves are listed in Table 2. They are sorted according to their occurrence times (with respect to the phases of the solar eclipse event). Within data, we detected also several waves before the solar eclipse events. We do not report on such waves here since they are very likely not related to the solar eclipse. Due to the fact that all three events occur during morning hours and because the performed measurements cover also sunrise hours, such waves probably originate from Solar Terminator movements. Hence, waves appearing before first contact of each eclipse event are not presented nor further discussed.

5.2 Gravity waves

Let us analyze, for each of the three solar eclipses, the first detected wave, propagating structures within the studied height range.
11 August 1999: Gravity wave 1. This is the wave chosen to illustrate the behaviors of the detection and characterization procedures described in Section 4 and in Fig. 4. Shortly after the first contact around 9h 20min UT, an upward propagating structure with period about 30 minutes and a downward traveling wave with period about 15 minutes are found. Fig. 5 reports the measurements obtained from the data characterizing these waves. One notices that the downward wave slightly precedes the upward moving wave (Fig. 5 (a)). Discontinuity between heights 190 km - 210 km in the wave time localization indicates the vertical size of the wave source region. Due to limitations of vertical ionospheric sounding method we cannot precisely decide whether these two waves are emitted from one source or from two distant sources located within region 190 km - 210 km. Parameters of the ionospheric plasma and neutral atmosphere at height 190 km differ from those at height 210 km. Hence, it may influence the resulting upward and downward propagating waves. A discontinuity is also clearly visible in the wave period (cf. Fig. 5 (b)), between the periods of the upward and downward moving structures, which indicates that we are observing two independent waves. Maximum amplitude of the upward wave is located around 240km (see Fig. 5 (c)). An important property of gravity waves lies in the fact that the phase propagates downward while the wave is moving upward or vice versa. Fig. 5 shows the wavevectors and the vertical components of the phase and packet velocities measured from the data. The positive sign of the packet velocity, together with the negative sign of the phase velocity, confirms that a gravity wave is found, that propagates upward from altitude 200km. Conversely, the gravity wave propagates downward below 200km. Fig. 6 shows all the wave parameters - wave vector, wavelength, phase and packet velocities, phase and energy angles - characterizing the propagations and derived from acoustic-gravity wave theory. The validation for the detection of a gravity wave is highlighted by the difference between energy (\(\alpha\)) and phase (\(\Phi\)) angle which is close to 90 degrees. Moreover, one notices that these waves propagates along directions close to the diagonals. Characteristic wavelength is found to be around 200 km.

In this case, as in most cases, we find an extremely satisfactory agreement between the z-components of the phase and packet velocities measured from the data and derived from disperse relation. This agreement takes into account Comment 2, made in Section 4.2 regarding the discrepancy between the measured quantity \(v^z_\phi\) and the z-component of the phase velocity \(v^z_{\phi,z}\). These agreements constitute clear confirmations of the validity of the detection of a gravity wave and a clear validation of the relevance of our combination of measurements made from data and characterization obtained from equations.

3 October 2005: Gravity wave 1. For the 3 October 2005 event, the first
detected wave structure occurs at 9h 12min, close to the eclipse maximum (see Fig. 7(a)). This gravity wave with period about 40 minutes (Fig. 7(b)) propagates upward between 155 km and 200 km (as seen from the positive packet velocity and negative phase velocities (cf. Fig. 7(e) and (f))). Wave amplitude maximum appears to be at height 180 km. Fig. 8 illustrates the complete propagation characteristics of the gravity wave. Its wavelength is found to be around 200 km (Fig. 8(d)). Comparison of the z-components of the phase and packet velocities (Fig. 8(b) and (c) shows high agreement between measured values and computed from dispersive relation. The fact that wave is propagating obliquely upward with angle \( \alpha \) close to 50 degrees indicates that this wave is moving from lower-laying atmosphere. Horizontal component of wave motion suggests that wave originates in distant location, possibly in the region with higher eclipse magnitude. It also explains, why the first gravity wave detected during annular eclipse event is seen as late as close to eclipse maximum.

29 March 2006: Gravity wave 1. For the 29 March 2006, inspection of the wavelet power spectrum indicates that a well developed structure is observed 14 minutes only after the first contact, cf. Fig. 9(a). This structure propagates through the lower part of the analyzed ionospheric region, from 160 km up to 205 km, with a period around 30 minutes (Fig. 9(b)). Wave reaches its maximum amplitude at height around 200 km (Fig. 9(c)). Fig. 9(d) shows the vertical component of the wave vector. The values of the z-components of the phase (negative) and packet (positive) velocities obtained from the data reveal that we observe an upward propagating gravity wave structure (cf. Fig. 9(e) and (f)). The complete set of propagation parameters is illustrated in Fig. 10(a)-(f). Fig. 10(b) shows an excellent agreement between estimated and computed vertical components of the phase velocity. The measured and computed vertical packet velocities do not match perfectly in the whole range (for reasons discussed in Section 4.2), however, there is a reasonable agreement with respect to sign and magnitude. Fig. 10(a) shows ll components of wave vector, resulting wavelength \( \lambda \) is plotted on Fig. 10(d). Values of angle \( \alpha \), that characterize energy progression, indicate that detected structure is moving obliquely upward from distant lower laying source.

Discussion. Equivalent analyses and plots for each of the detected structures mentioned in Table 2 are available upon request or can be found at http://www.ufa.cas.cz/html/climaero/sauli.html. The study of these structures yields the following comments.

The gravity wave activity increases after a notably larger delay for the annular solar eclipse compared to the total solar eclipses: waves are found during the maximum phase of the eclipse only for the former while they occur during the initial phase for the latter. This discrepancy in gravity waves generation/occurrence can likely be explained by differences in the terrestrial Atmo-
sphere cooling: the border between sunlit and eclipsed region is much sharper in the case of total eclipse.

Analyzing wave propagations, we observe predominantly upward propagating structures. The wave structure that propagate upward and downward from the source region located around 200 km height consists of an exceptional case related to the solar eclipse of August 11, 1999. As it has been proposed by Liu et al. 1998 changes of temperatures, and variations of the height of the transition level for the loss coefficient and the height of the peak of electron production may be effective mechanism for wave generation in-situ, at ionospheric heights, during solar eclipse even and such emitted waves may propagate upward and downward from source region. Such a situation does not repeat in any of the two other analyzed events, when the coverage of the solar disk is much lower.

This finding might reflect that we mostly observe signatures of the shock wave or structures propagating from further distance during the two later events. Supersonic motion of the moon shadow, that forms shock structure (Chimonas and Hines 1970, Altadill et al. 2001b) and abrupt of solar radiation with consequent interruption of ozone heating in the lower-laying atmosphere (Fritts and Luo 1993) may form/launch waves that reach ionospheric heights.

Values of the energy propagation angles $\alpha$ (Fig. 6, 8 (f) and Fig. 10 (f)), of all the detected gravity waves, indicate an oblique propagation direction. The difference between energy ($\alpha$) and phase ($\Phi$) angles, close to 90 degrees, confirm the gravity wave nature of the detected waves.

5.3 Acoustic wave

During the initial phase of the August, 11, 1999 event, we found a line of maxima existing over a large range of heights $z$ and with a period ranging from 3 to 4 minutes (cf. Fig. 11). The identical signs of packet and phase velocities (cf. Fig. 12), together with the propagation period range suggest that this is an acoustic wave. Fig. 13 shows the characteristics of the wave as derived from the model. For this wave, while the agreement between the measured and derived $z$-components of the phase velocity is very satisfactory, this is not the case for the packet velocity (cf. Fig. 13(e) and (f)). This can be easily understood as the measured packet velocity involves taking an empirical double derivative, at periods (3 to 4 minutes) which are extremely close to the sampling period (1 min): this is hence a ill-conditioned numerical operation. This points out a major difficulty in detecting and characterizing acoustic waves: detecting waves, whose periods are of the order of a few minutes, from data collected at sampling rates commonly used, above 1 min, is barely possible and even meaningless; a relevant tracking of acoustic waves requires the use of sampling periods well below the minute. However, in the 1999 event, the combined use of a wavelet-based time-frequency representation, together with the exceptionally
low 1-min sampling period, enables us to unambiguously detect an acoustic wave, which, as far as we know, has very rarely been achieved. Moreover, the use of the equation-based characterization that we proposed here allows us to accurately identify its propagating parameters. For instance, we find that the energy and phase angles are close one from the other as opposed to what is found for gravity waves. For this wave, we also find that the value of the energy angle indicate an oblique propagation of the wave. Hence, despite this sampling rate issue, the characterization of the acoustic wave proposed proves valid and gives satisfactory results.

6 Conclusions

In the present contribution, we showed that, taking advantage of the excellent joint time and frequency localization properties of the wavelet transform, we are able to detect and characterize wave structures. The detection relies on the identification of a collection of local modulus maxima, occurring simultaneously over a continuous range of heights. From the (derivation of the) phase of the complex wavelet coefficients, we managed to measure the $z-$ components of the wave, phase and packet velocity vectors. Furthermore, making use of the acoustic-gravity wave propagation equations, we managed to fully characterize the corresponding propagating parameters. This characterization part only relies on the use of the measured $z$-component of the wave vector. A key point in our approach lies in the use of sequences of vertical profiles of electron concentration and in the derivation of vertical and horizontal characteristics of the propagating pulse. Making use of this tools, we were able to identify numerous gravity waves and one acoustic wave. Hence, our analysis confirms the occurrence and production/enhancement of acoustic-gravity waves, at ionospheric heights, during solar eclipses. Notably, we observed that for high magnitude of total solar eclipses AGW occur extremely quickly after the beginning of the event. Also, we highlighted the difficulties in acoustic-wave detection and characterization as well as the need for much higher sampling rate when acoustic waves are targeted.

We believe that the use of the toolbox proposed here brings significant improvements and benefits with respect to efficient wave detections and can hence be easily used by the Ionospheric community.
References


[31] Row, R.V., 1967 Acoustic-gravity Waves in Upper Atmosphere Due to
a Nuclear and an Earthquake. Journal of Geophysical Research, 72 (5): 1599&.


Fig. 1. **3-hour Kp index.** Geomagnetic activity during three solar eclipses events. The grey shaded areas indicate occurrences time of the eclipses.
Fig. 2. **Electron concentrations.** Variations of the electron concentrations (at fixed heights 155 km - 255 km) as a function of time for eclipse 1999, 11 August (a) and (b); 2005, 3 October (c) and (d); and 2006, 29 March (e) and (f), (left column) and the corresponding fluctuations left after removing the main trend (right column). For the right column, an arbitrary shift proportional to $z$ is added so that each time series can be seen. The grey shaded areas indicate occurrences time of the eclipses.
Fig. 3. **Paul Wavelets.** Left: real, imaginary and modulus of $N = 4$ Paul complex wavelet, as a function of time, in hours (arbitrarily centered around $t = 0$); Right, modulus of the Fourier transform of the corresponding wavelets, as a function of Period (in min); For analysis scales corresponding to periods $P = 5\text{min}$ (top) and $P = 30\text{min}$ (bottom). The time and Period widths of these wavelets match the sizes of the energy concentrations packets that can be observed in the scalograms in Fig. 4, for $P = 30\text{min}$ and Fig. 11 for $P = 5\text{min}$. 
Fig. 4. Scalograms, 11 August 1999. Examples of scalogram plots at three consecutive heights 220km (a) and (b); 215 km (c) and (d); and 210 km (e) and (f). In left column, the time range is selected to cover the initial solar eclipse phase. Local maxima line are marked with "•". Around 9h15min UT, there exists a well developed line of local maxima, marked with a "▲", that coincide in time and period over a significant range of heights z. Panels (b), (d) and (f) represent the focus on this maxima in the period range 31-45 minutes between 9h UT and 9h30min UT.
Fig. 5. **11 August 1999, GW1: Detection.** Time location (a), period (b) and amplitude (c) of the detected wave, with the vertical components of the wave vector $k_z$ (d), phase velocity $v_{\phi}^{(z)}$ (e) and packet velocity $v_{(p,z)}$ (f).
Fig. 6. **11 August 1999, GW1: Characterization.** Wave parameters computed from the AGW theory: Wavevector (a), Phase velocity (b), packet velocity (c), wave number (d), energy (e) and phase (f) angles. For the vectors of first row, the '□' correspond to the measured (black) and computed (white) $z$-components, the '◦' correspond to the horizontal components while the '▽' are related to the modulus. Note that for the Phase velocity (b), the agreement between the measured (black '□') and the computed (white '□') $z$-component is so satisfactory that the latter can hardly be seen.
Fig. 7. 3 October 2005, GW1: Detection. Same legend as Fig. 5.

Fig. 8. 3 October 2005, GW1: Characterization. Same legend as Fig. 6.
Fig. 9. 29 March 2006, GW1: Detection. Same legend as Fig. 5.

Fig. 10. 29 March 2006, GW1: Characterization. Same legend as Fig. 5.
Fig. 11. **Scalograms, 11 August 1999.** Examples of three scalogram plots at three consecutive heights 200km (a) and (b); 195 km (c) and (d); and 190 km (e) and (f). In left column, the time range is selected to cover the initial solar eclipse phase. Local maxima line are marked with "•". Between 10h UT and 10h20min UT, there is well developed local maxima, marked with a "▲", that coincides in time and period over a significant range of heights $z$s. Panels (b), (d) and (f) represent the focus on this maxima in the acoustic mode period range 2-6 minutes around 10H15min UT.
Fig. 12. 11 August 1999, AW1: Detection. Same legend as Fig. 5.

Fig. 13. 11 August 1999, AW1: Detection. Same legend as Fig. 6.
<table>
<thead>
<tr>
<th>Event</th>
<th>First contact</th>
<th>Fourth contact</th>
<th>Maximum</th>
<th>Magnitude</th>
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<tr>
<td>11 Aug 1999</td>
<td>09:22</td>
<td>12:04</td>
<td>10:42</td>
<td>0.952</td>
</tr>
<tr>
<td>3 Oct 2005</td>
<td>08:01</td>
<td>10:32</td>
<td>09:15</td>
<td>0.539</td>
</tr>
<tr>
<td>29 Mar 2006</td>
<td>09:46</td>
<td>11:50</td>
<td>10:48</td>
<td>0.486</td>
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</tbody>
</table>

Table 1

Parameters of the solar eclipse events. Parameters as observed above Prühonice ionospheric station, (According to NASA database, time is given in UT).
<table>
<thead>
<tr>
<th>Event</th>
<th>name</th>
<th>Period</th>
<th>Occurrence</th>
<th>Propagation</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 August 1999</td>
<td>GW1</td>
<td>15 min, 30 min</td>
<td>Initial</td>
<td>upward/downward</td>
</tr>
<tr>
<td></td>
<td>AW1</td>
<td>3-4 min</td>
<td>Initial</td>
<td>upward/downward</td>
</tr>
<tr>
<td></td>
<td>GW2</td>
<td>22 min</td>
<td>After</td>
<td>upward</td>
</tr>
<tr>
<td>3 October 2005</td>
<td>GW1</td>
<td>43 min</td>
<td>Maximum</td>
<td>upward</td>
</tr>
<tr>
<td></td>
<td>GW2</td>
<td>20 min</td>
<td>Recovery</td>
<td>upward</td>
</tr>
<tr>
<td></td>
<td>GW3</td>
<td>65 min</td>
<td>After</td>
<td>upward</td>
</tr>
<tr>
<td></td>
<td>GW4</td>
<td>30 min</td>
<td>After</td>
<td>upward</td>
</tr>
<tr>
<td></td>
<td>GW5</td>
<td>32 min</td>
<td>After</td>
<td>upward</td>
</tr>
<tr>
<td></td>
<td>GW6</td>
<td>22 min</td>
<td>After</td>
<td>upward</td>
</tr>
<tr>
<td>29 March 2006</td>
<td>GW1</td>
<td>30 min</td>
<td>Initial</td>
<td>upward</td>
</tr>
<tr>
<td></td>
<td>GW2</td>
<td>40 min</td>
<td>After</td>
<td>upward</td>
</tr>
</tbody>
</table>

Table 2

**Detected waves.** List of the detected and analyzed waves with occurrence period and time (with respect to the eclipse phase). Period of the wave denotes dominant period of the structure.

Fig. 1. **1 August 1999, GW2: Detection.** Same legend as Fig. 5. Morlet wavelet, $\mu = 6$. 
Fig. 2. 1 August 1999, GW2: Modeling. Same legend as Fig. 6. Morlet wavelet, $\mu = 6$.

Fig. 3. 3 October 2005, GW2: Detection. Same legend as Fig. 5. Morlet wavelet, $\mu = 4$. 
Fig. 4. 3 October 2005, GW2: Modeling. Same legend as Fig. 6. Morlet wavelet, $\mu = 4$.

Fig. 5. 3 October 2005, GW3: Detection. Same legend as Fig. 5. Morlet wavelet, $\mu = 6$. 
Fig. 6. 3 October 2005, GW3: Modeling. Same legend as Fig. 6. Morlet wavelet, $\mu = 6$.

Fig. 7. 3 October 2005, GW4: Detection. Same legend as Fig. 5. Morlet wavelet, $\mu = 2$. 

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Fig. 8. 3 October 2005, GW4: Modeling. Same legend as Fig. 6. Morlet wavelet, $\mu = 2$.

Fig. 9. 3 October 2005, GW5: Detection. Same legend as Fig. 5. Morlet wavelet, $\mu = 4$. 
Fig. 10. 3 October 2005, GW5: Modeling. Same legend as Fig. 6. Morlet wavelet, $\mu = 4$.

Fig. 11. 3 October 2005, GW6: Detection. Same legend as Fig. 5. Morlet wavelet, $\mu = 2$. 
Fig. .12. 3 October 2005, GW6: Modeling. Same legend as Fig. 6. Morlet wavelet, $\mu = 2$.

Fig. .13. 29 March 2006, GW2: Detection. Same legend as Fig. 5. Paul wavelet, $\mu = 6$. 
Fig. 14. **29 March 2006, GW2: Modeling.** Same legend as Fig. 6. Paul wavelet, \( \mu = 6 \).